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He's frequency formulation for the relativistic harmonic oscillator

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ABSTRACT

In this paper, He's frequency formulation is used to solve the nonlinear differential equation that governs the oscillation of the relativistic oscillator. The obtained solution with high accuracy shows that He's frequency formulation is a simple but promising method without any requirement for advanced calculus.

Consider the following nonlinear differential equation of a relativistic oscillator [1]

$$\frac{\mathbf{d}^2\boldsymbol{u}}{\mathbf{d}\boldsymbol{t}^2}+\frac{\boldsymbol{u}}{\sqrt{1+\boldsymbol{u}^2}}=0 \tag{1}$$

with initial conditions

$$\boldsymbol{u}(0)=\boldsymbol{B},\qquad u'(0)=0. \tag{2}$$

According to He's frequency formulation [2–5], we have

$$\omega^2=\frac{\omega_1^2R_2(t_2)-\omega_2^2R_1(t_1)}{R_2(t_2)-R_1(t_1)} \tag{3}$$

where t_1 and t_2 are location points. Generally we set [6]

$$t_1=\frac{T_1}{12},\qquad t_2=\frac{T_2}{12} \tag{4}$$

where T_1 and T_2 are periods of the trial solutions $u_1(t)=B\cos t$ and $u_2(t)=B\cos\omega t$, respectively, which lead to the following residuals:

$$\boldsymbol{R}_1(\boldsymbol{t}_1)=-\boldsymbol{B}\cos\boldsymbol{t}_1+\frac{\boldsymbol{B}\cos\boldsymbol{t}_1}{\sqrt{1+\boldsymbol{B}^2\cos^2\boldsymbol{t}_1}}, \tag{5}$$

$$\boldsymbol{R}_2(\boldsymbol{t}_2)=-\boldsymbol{B}\boldsymbol{\omega}^2\cos\boldsymbol{\omega}\boldsymbol{t}_2+\frac{\boldsymbol{B}\cos\boldsymbol{\omega}\boldsymbol{t}_2}{\sqrt{1+\boldsymbol{B}^2\cos^2\boldsymbol{\omega}\boldsymbol{t}_2}}. \tag{6}$$

Using Eq. (3), we obtain:

$$\omega^2=\frac{\omega_1^2R_2(t_2)-\omega_2^2R_1(t_1)}{R_2(t_2)-R_1(t_1)}=\frac{-B\omega^2\cos\frac{\omega T_2}{12}+\frac{B\cos\frac{\omega T_2}{12}}{\sqrt{1+B^2\cos^2\frac{\omega T_2}{12}}}-\omega^2\left(-B\cos\frac{T_1}{12}+\frac{B\cos\frac{T_1}{12}}{\sqrt{1+B^2\cos^2\frac{T_1}{12}}}\right)}{-B\omega^2\cos\frac{\omega T_2}{12}+\frac{B\cos\frac{\omega T_2}{12}}{\sqrt{1+B^2\cos^2\frac{\omega T_2}{12}}}-\left(-B\cos\frac{T_1}{12}+\frac{B\cos\frac{T_1}{12}}{\sqrt{1+B^2\cos^2\frac{T_1}{12}}}\right)}=\frac{1}{\sqrt{1+\frac{3}{4}B^2}} \tag{7}$$

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which is the same as the first analytical approximate frequency obtained by Beléndez [1].

Hereby He's frequency formulation is proved to be a powerful mathematical tool in the search for periodic solutions of nonlinear oscillators without the requirement of perturbation or linearization; it is simple, straightforward and effective.

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